

# Reduced Model of Engine Air Path System Using a LPV Approach

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This paper describes a reduced model of an engine air path composed of a supercharger with throttle valve. This “industry-oriented” model is intended for control purposes.

The air-path system dynamics are governed by the Saint-Venant equations, difficult to handle in control synthesis due to nonlinear properties (states - control inputs coupling, nonlinear functions). To take account the intrinsic nonlinear properties of the air-path model in control purpose, a Linear Parameter Varying (LPV) state space representation is proposed. The model is simplified considering that slow dynamics are constant, allowing to reduce the number of time varying parameters, and, therefore, the calibration time for control synthesis.

The reduced LPV model is compared with the nonlinear one, and validated on air path benchmark data, showing that the model is representative enough of the system, and suitable for advanced and robust controller synthesis.

Topics / Modeling and simulations, State estimation

## 1. INTRODUCTION

Nowadays, severe norms impose on car manufacturers strict pollutant emission regulations (EURO normative in Europe). To meet future pollutant emission standards while responding to customer request of performances (fuel consumption reduction with the same engine power), new technologies have to be developed, incorporating more sophisticated engine control strategies. A promising way to achieve this goal is the engine downsizing, reducing fuel consumption without degrading the delivered power. Additional systems such as supercharger or turbocharger, electronic actuators (for example, throttle valve or variable valve timing) complicate the air path system of downsized engines, where more and more advanced control techniques have to be developed.

During the last decade, the air path system has become a crucial part in engine development. Indeed, air path control is strongly linked to engine performances, since the torque is directly related to the injected air quantity, and also to engine pollutant emission control. The objective of the air path control is to provide the driver's torque request while injecting the accurate amount of air mass flow to ensure pollutant emission control and power requested.

The air-path system dynamics are governed by the

Saint-Venant equation, difficult to handle in control synthesis due to its non-linear properties (states - control inputs coupling, nonlinear functions). To manage these nonlinear equations, many nonlinear techniques exist using several methods: input to state linearization controller as in [1] or [2] and predictive control as in [3]. Authors in [1] present a strategy consisting of constrained motion planning and feedback linearization but properties of the closed loop system (convergence, stability, constraints) are harder to prove. In [4], a Takagi-Sugeno (TS) model is considered to develop a switching control strategy.

This paper presents a LPV model of a spark ignition (SI) engine air-path system composed of a supercharger with throttle valve. The paper is organized as follows. In section 2, the nonlinear model of the air path system is presented. Then, the model is simplified considering that slow dynamics are constant, reducing the number of time varying parameters, and therefore the calibration time for control. In section 4, a Linear Parameter Varying (LPV) state space representation is proposed from the Saint-Venant equations. Using a LPV representation, the intrinsic nonlinear properties of air-path model are well represented, and can be taken into account in a model based control with a LPV controller. Section 5 is devoted to the LPV model simulation results. Validation is done on experimental results obtained from an air path test bench.

## 2. SPARK IGNITION ENGINE AIR PATH SYSTEM

The studied system is a spark ignition engine air path composed of a throttle valve and a supercharger, in combination with a bypass valve (Fig. 1).

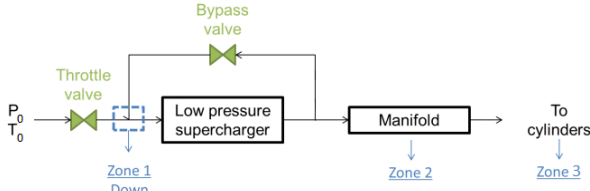


Fig. 1 : Studied air-path system

The intake mass flow varies according to the angular variation of the throttle valve. The supercharger speed is proportional to the engine one. To avoid surge situation, the supercharger is also equipped with a bypass valve, which is actuated when intake supercharger pressure is less than the one at his exhaust. From control point of view, the main difficulty in such an air path structure is to control the engine intake pressure while managing the supercharger dynamics since the supercharger is located after the throttle valve. Therefore, the air path modeling is a critical step for control since the model has to be enough accurate on the internal dynamics and reduced for industrial application.

The following model is obtained from energy conservation law and Saint-Venant equation which gives the mass flow rate through a section (for more details on the equations, see [5], [6]). The air-path system is modeled by the following equations:

Notations	
$Q_i$	Mass flow rate ( $\text{kg.s}^{-1}$ )
$P_i$	Pressure (Pa)
$T_i$	Temperature (K)
$P_0$	Ambient pressure (Pa)
$T_0$	Ambient temp. (K)
$T_{wall}$	Wall temperature, $T_{wall}=T_0$
$X_c$	Critical pressure ratio
$S_i$	Effective section area ( $\text{m}^2$ )
$V_i$	Volume ( $\text{m}^3$ )
$Cd_i$	Discharge coefficients
$\theta_i$	Opening valve angle
$h_1, h_2$	Thermal exchange constants
$C_p$	Heat capacities ratio
$r$	Gas constant
$\gamma$	Heat capacities ratio
$\beta$	Supercharger efficiency
$1, 2, 3$	Zones after throttle, in the manifold, in cylinder
$thr$	Throttle
$sc$	Supercharger
$byp$	Bypass
$man$	Manifold
$cyl$	Intake cylinder

**Air mass flows** (St-Venant equations)

$$Q_{thr} = Cd_{thr} S_{thr} (\theta_{thr}) \frac{P_0}{\sqrt{rT_0}} X_1^{\frac{1}{\gamma}} \sqrt{\frac{2\gamma}{\gamma-1} \left(1 - X_1^{\frac{\gamma-1}{\gamma}}\right)}$$

$$Q_{byp} = Cd_{byp} S_{byp} (\theta_{byp}) \frac{P_2}{\sqrt{rT_2}} X_2^{\frac{1}{\gamma}} \sqrt{\frac{2\gamma}{\gamma-1} \left(1 - X_2^{\frac{\gamma-1}{\gamma}}\right)}$$

$$Q_{cyl} = Cd_{cyl} S_{cyl} \frac{P_2}{\sqrt{rT_2}} X_3^{\frac{1}{\gamma}} \sqrt{\frac{2\gamma}{\gamma-1} \left(1 - X_3^{\frac{\gamma-1}{\gamma}}\right)}$$

$Q_{sc}$  is obtained by a 2D lookup table.

(1)

For each  $i=1,2,3$

$$\text{If, } X_i > X_c: \quad X_1 = \frac{P_1}{P_0}, \quad X_2 = \frac{P_1}{P_2}, \quad X_3 = \frac{P_{cyl}}{P_2}$$

$$\text{Else, } X_i \leq X_c: \quad X_i = X_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

**Pressure dynamics**

$$\dot{P}_1 = \frac{rT_1}{V_1} (Q_{thr} - Q_{sc} + Q_{byp})$$

$$\dot{P}_2 = \frac{rT_2}{V_2} (-Q_{byp} - Q_{cyl} + Q_{sc})$$

(2)

$P_{cyl}$  is assumed to be equal to  $P_0$  (burnt gas are all exhausted during air admission).

**Temperature dynamics**

$$\dot{T}_1 = \frac{rT_1}{P_1 V_1} \left( T_0 Q_{thr} - T_1 Q_{sc} + T_2 Q_{byp} - \frac{h_1 S_1}{C_p} (T_1 - T_{wall}) \right)$$

$$\dot{T}_2 = \frac{rT_2}{P_2 V_2} \left( T_1 \beta_e Q_{sc} - T_2 Q_{cyl} - T_2 Q_{byp} - \frac{h_2 S_2}{C_p} (T_2 - T_{wall}) \right)$$

$$\text{With } \beta_e = 1 + \frac{1}{\beta} \left( \left( \frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right).$$

(3)

This model is composed of 4 states ( $P_1, P_2, T_1, T_2$ ) and 5 parameters ( $Cd_{thr}, Cd_{byp}, Cd_{cyl}, h_1, h_2$ ) to be identified. The temperature equations take into account the heat losses through the walls. However, studies of the GT Power simulations and experiment data have shown that back flows along the air-path system can be neglected. So, Saint-Venant equations can be simplified.

## 3. AIR PATH MODEL REDUCTION

In order to use a model based control, this model is now reduced. This operation leads to a reduction of the number of parameters to identify, and therefore saves calibration time. By assuming that the air temperature after the throttle valve equals the ambient temperature, and that the temperature variations described in Eq. 3 are slow compared to the pressure variations, the previous temperature equations become:

$$\dot{T}_1 = \dot{T}_2 = 0$$

These reductions lead to:

$$\begin{aligned} T_1 &= T_0 = 300 \text{ K} \\ T_2 &= \frac{T_1 \beta_e + \frac{h_2 S_2}{c_p} T_{\text{wall}}}{\frac{h_2 S_2}{c_p} + Q_{\text{byp}} + Q_{\text{cyl}}} \end{aligned} \quad (4)$$

The reduced model is then composed of 2 states ( $x = [P_1 \ P_2]$ ) with 4 parameters ( $Cd_{thr}$ ,  $Cd_{byp}$ ,  $A_{cyl} = Cd_{cyl} S_{cyl}$ ,  $h_2$ ) to identify.

#### 4. LPV REPRESENTATION APPROACH

This section presents the proposed LPV representation. LPV representation is a natural extension of Linear Time Invariant (LTI) systems. A LPV dynamical system has the same form as a LTI one, where nonlinearities are taken into account by the varying parameters. Therefore, the matrices of the linear representation depend on the varying parameters. For further into the subject, [7] is a complete reference on the LPV system control.

In the following,  $x$  denotes the state vector,  $y$  the vector of measurements with  $y=x$ ,  $u$  the control inputs and  $\rho_{1,2}$ , the varying parameters.

Let,

$$\begin{aligned} \rho_1(x) &= \left( \frac{x_1}{P_0} \right)^{\frac{1}{\gamma}} \sqrt{\frac{2\gamma}{\gamma-1} \left( 1 - \left( \frac{x_1}{P_0} \right)^{\frac{\gamma-1}{\gamma}} \right)} \\ \rho_2(x) &= \left( \frac{x_1}{x_2} \right)^{\frac{1}{\gamma}} \sqrt{\frac{2\gamma}{\gamma-1} \left( 1 - \left( \frac{x_1}{x_2} \right)^{\frac{\gamma-1}{\gamma}} \right)} \end{aligned}$$

And,

$$\begin{aligned} u_{\text{int}1} &= S_{\text{thr}}(\theta_{\text{thr}}) \\ u_{\text{int}2} &= Q_{\text{byp}} - Q_{\text{sc}} \end{aligned}$$

The control inputs  $u = [\theta_{thr}, \theta_{byp}]$  are obtained by inverting the expressions of the intermediate control inputs  $u_{\text{int}1}$  and  $u_{\text{int}2}$ .

The previous model can be written in the following quasi-LPV form:

$$\begin{aligned} \dot{x} &= A(\rho(x))x + B(\rho(x))u \\ y &= x \end{aligned} \quad (5)$$

With,

$$\begin{aligned} A &= \begin{pmatrix} 0 & 0 \\ 0 & -\frac{\sqrt{rT_2}}{V_2} Cd_{cyl} S_{cyl} \rho_2 \end{pmatrix} \\ B &= \begin{pmatrix} \frac{\sqrt{rT_1}}{V_1} P_0 \rho_1 & -\frac{rT_1}{V_1} \\ 0 & \frac{rT_2}{V_2} \end{pmatrix} \end{aligned}$$

It should be noticed that this system is quasi-LPV since the parameters  $\rho_{1,2}$  depend on the state vector. They are measured and bounded in the convex set  $[\underline{\rho}_{1,2}, \bar{\rho}_{1,2}]$  so a polytopic approach can be used for control purpose, with  $\underline{\rho}_{1,2} = X_c$  and  $\bar{\rho}_{1,2} = 0.99$  (the maximum of the parameter is considered different from 1 to avoid numerical problem).

For control use, this affine form can be written under the form of a polytopic system composed of  $N=4$  vertices formed by the bounds of the varying parameters  $\rho_{1,2}(x)$ . The polytopic system is written in the following form:

$$\begin{bmatrix} A(\rho) & B(\rho) \\ C(\rho) & D(\rho) \end{bmatrix} = \sum_{i=1}^N \alpha_i(\rho(x)) \begin{bmatrix} A(\omega_i) & B(\omega_i) \\ C(\omega_i) & D(\omega_i) \end{bmatrix}$$

Where  $\omega_i$  defines each vertex of the polytope composed by the minimum and maximum of each varying parameters  $\rho_{1,2}(x)$ .

The scheduling function  $\alpha_i(\rho_{1,2}(x))$  is defined as,

$$\alpha_i(\rho(x)) = \frac{\prod_{k=1}^I |\rho_k - C(\omega_i)_k|}{\prod_{k=1}^I (\bar{\rho}_k - \underline{\rho}_k)}, \quad i=1, \dots, N$$

and satisfies the following convex properties:

$$\begin{aligned} \sum_{i=1}^n \alpha_i(\rho_{1,2}(x)) &= 1 \\ 0 &\leq \alpha_i(\rho_{1,2}(x)) \leq 1 \end{aligned}$$

The  $k^{\text{th}}$  component of the vector  $C(\omega_i)$ , denoted  $C(\omega_i)_k$  is defined as,

$$\begin{aligned} C(\omega_i)_k &:= \begin{cases} \rho_k & \text{if } (\omega_i)_k = \underline{\rho}_k, \rho_k \\ \bar{\rho}_k & \text{otherwise} \end{cases} \end{aligned}$$

The LPV representation is advantageous since linear control approaches can be considered while keeping all the intrinsic nonlinear properties of the model. Moreover, the pressures and air mass flow along the air path system are low, which makes robust control essential. A LPV controller provides an adaptive and robust control since  $H_\infty$  criteria can be added in controller synthesis. Authors in [8] present an example of a LPV system design.

#### 5. EXPERIMENTAL RESULTS

This section presents the validation of the identified reduced LPV model. Identification was done on one set of the experimental data. In order to find the calibration parameters, nonlinear least square techniques were used in this work. Model validation is done on a different set of air path test bench data. Experimental data from the air path test bench have been obtained on several throttle opening values and different engine speed. The throttle valve opens at 10%, 30% and 100% (Fig. 2) with the engine speed varying between 1000 rpm and

4000 rpm. The ambient temperature was kept at 300 K. The pressures, temperatures and intake mass flow were measured on the bench.

The studied system is composed of a throttle valve, a Rotrex supercharger with a bypass valve to handle surge points and a rotating disk to simulate the outlet ports of the air path system.

In the following, only results obtained with an engine speed of 2000 and 4000 rpm with throttle variation and, results of an operating point next to surge limit are shown.

**Case 1: Engine speed: 2000 rpm, Throttle valve opening: 10, 30 and 100%, Bypass valve opening: 0%.**

Fig. 2 shows the opening variations of throttle and bypass valves during the experiment. LPV model simulation results show a good correlation with the experiment data. The modeling error is kept small (less than 5%). Error increases when throttle valve variations reach the opening limits. Indeed, errors on pressures values are observed when the opening angle equals 10% and 100%, while the obtained calibration gives good results at an 30% opening angle. However, air mass flow is well simulated for the three opening angles (Fig. 5).

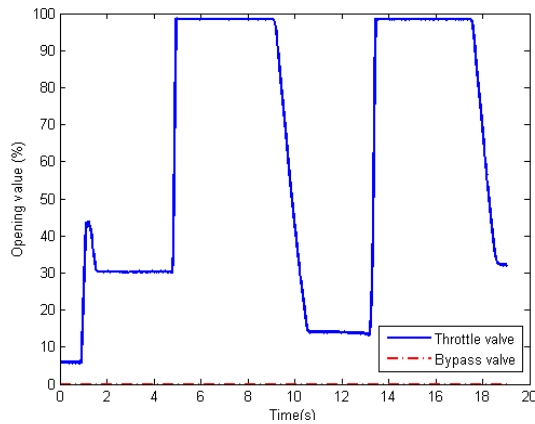


Fig. 2: Valves opening variations

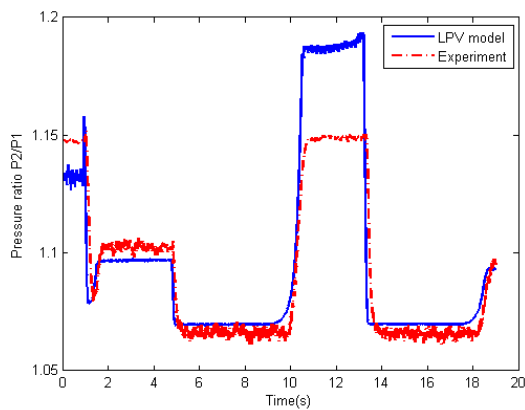


Fig. 3: Pressure ratio comparison of LPV (solid blue) and experimental data (dashed red)

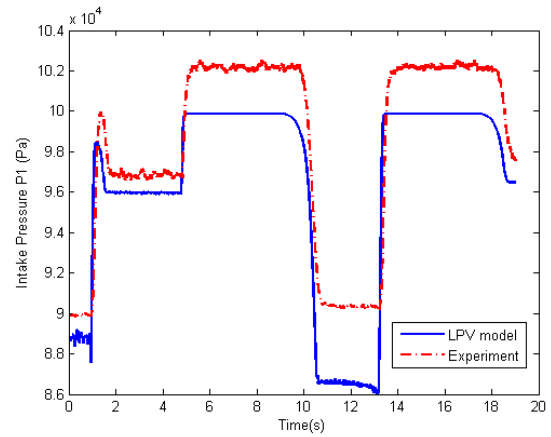


Fig. 4: Intake pressure comparison of LPV (solid blue) and experimental data (dashed red)

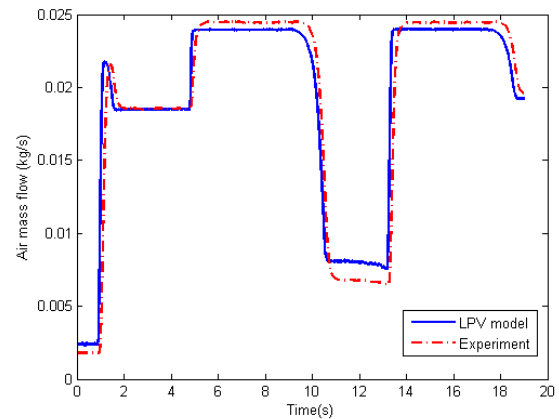


Fig. 5: Air mass flow comparison of LPV (solid blue) and experimental data (dashed red)

**Case 2: Engine speed: 4000 rpm, Throttle valve opening: 10, 30 and 100%, Bypass valve opening: 0%.**

The following results show again that modeling errors on pressures remain small (lower to 5%) (Fig. 7, 8). Contrary to the previous results, errors are present on the three throttle opening values. However, very good results are obtained for the intake air mass flow (Fig. 9).

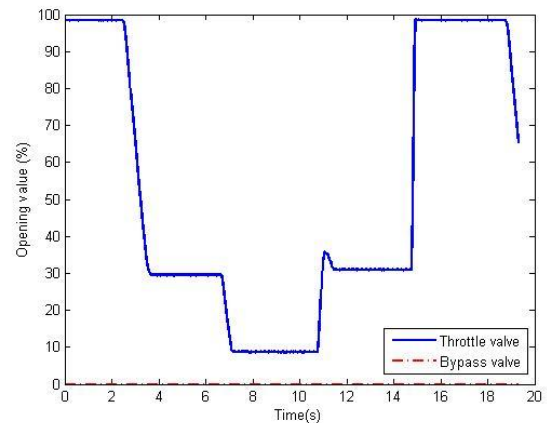


Fig. 6: Throttle and bypass valves opening variations

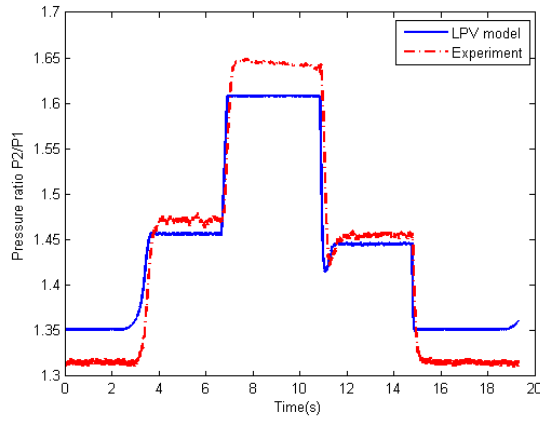


Fig. 7 : Comparison of the modeled and experimental pressure ratio

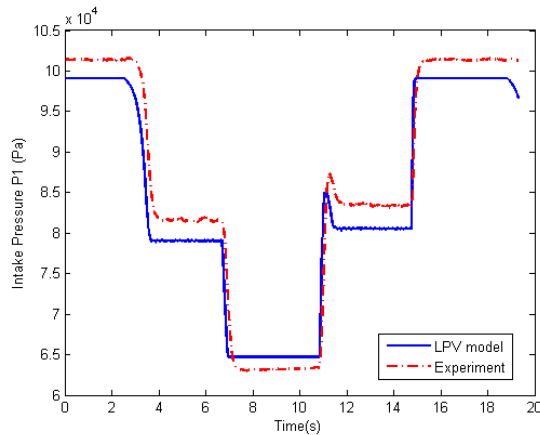


Fig. 8 : Comparison of the modeled and experimental intake pressure  $P_1$

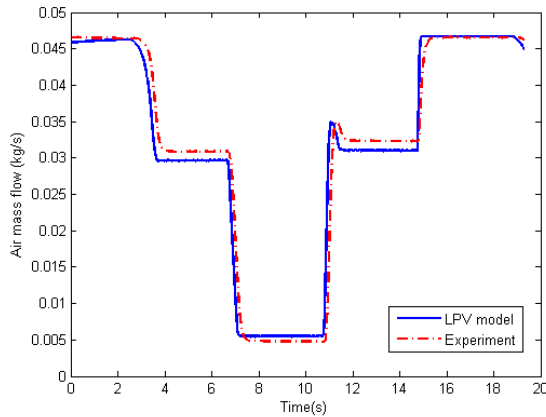


Fig. 9 : Comparison between LPV model and experiment intake air mass flow

**Case 3: Engine speed: 4500 rpm, Throttle valve opening: 10%, Bypass valve opening: 0 and 40%.**

In this specific case, experiment has been done on an operating point next to surge limits at an engine speed of 4500 rpm. Throttle valve was kept weakly opened (at

10%) while bypass valve opening varied from 0 to 40% (Fig. 10).

The obtained results fit quite well the experiment data. Again, error modeling on pressures exists but remains small (Fig. 11, 12). Air mass flow simulation (Fig. 13) gives good results where experiment and LPV model data overlap.

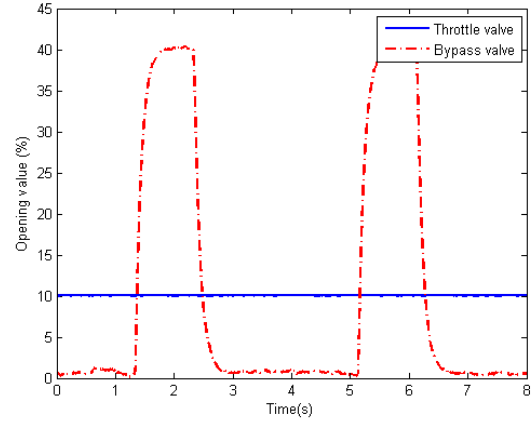


Fig. 10: Throttle and bypass valves opening variations

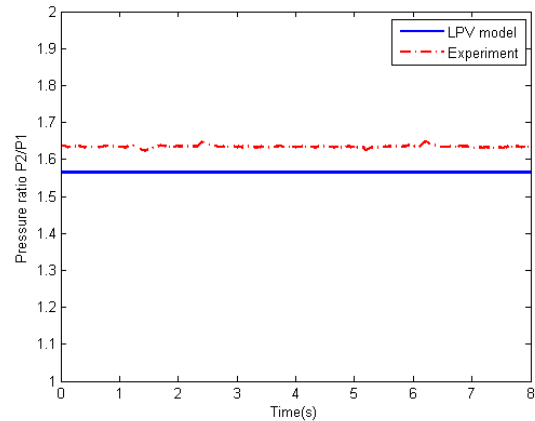


Fig. 11: Comparison of the modeled and experimental pressure ratio

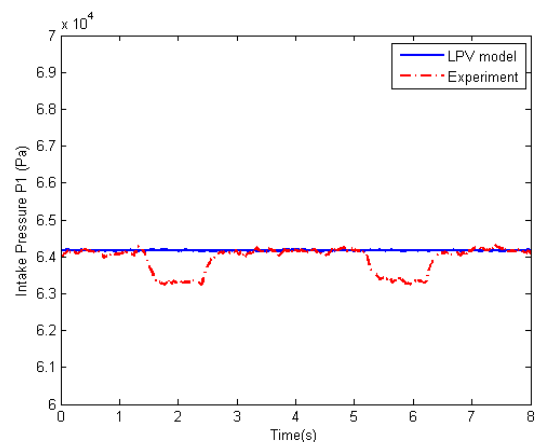


Fig. 12: Comparison of the modeled and experimental intake pressure  $P_1$

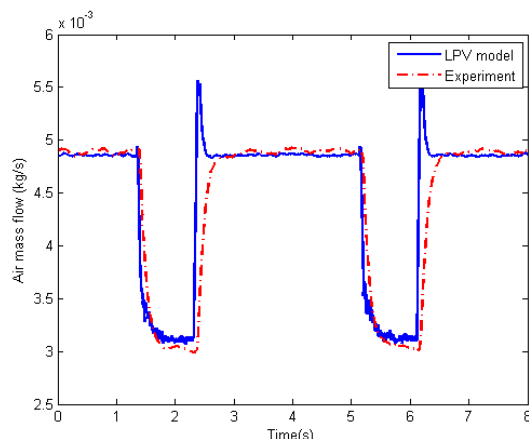


Fig. 13: Comparison between LPV model and experiment intake air mass flow

The LPV model is quite representative of the real air path dynamics. The simulation results show that the model is suitable for advanced controller synthesis. Modeling error remains small for pressures values (lower than 5%). Very good results are obtained for the air mass flow where simulations of LPV model fit well the experiment data. Error modeling can be taken into account by the LPV controller by adding this uncertainty in the model and, rejected by using a  $H_\infty$  criterion to obtain a robust LPV controller.

## 6. CONCLUSION

The main purpose of this article is to show that a LPV approach is possible on such a system. This LPV model is suitable for industry application because it provides calibration time savings since only four parameters have to be identified. The LPV approach is efficient to take into account the nonlinearities and uncertainties. Also LPV controllers are self-scheduled by the measured parameters leading to a simple controller structure (combination of LPV controllers), easy to be implemented. With this LPV model, robust multivariable controller for the presented air path system is possible and this model is well suited for industrial applications thank to the simplicity of its form.

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